

Baryon-Pion Scattering in the $1/N_c$ Expansion: Tree Diagram Cancellations

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Abstract

Tree amplitudes for baryon-pion scattering are studied in the $1/N_c$ expansion. Generalized large- N_c consistency conditions are obtained to all orders in baryon mass splittings. For baryons with spin $J \sim \mathcal{O}(1)$, the leading order in N_c tree amplitudes can be evaluated keeping only terms up to a given finite order in baryon mass splittings.

I. INTRODUCTION

The study of baryons in the $1/N_c$ expansion has led to significant theoretical progress in understanding the spin-flavor structure of QCD baryons (See Ref. [1] for a recent review). In the large- N_c limit, there exists a spin-flavor symmetry for baryons [2–4], and baryons form irreducible representations of the spin-flavor symmetry. For N_c large but finite, these irreducible representations contain baryons with spins ranging from $J \sim \mathcal{O}(1)$ to $J \sim \mathcal{O}(N_c)$. Important symmetry relations can be derived for baryons with spin $J \sim \mathcal{O}(1)$.

Dashen, Jenkins and Manohar [4] showed that large N_c power counting rules for multipion–baryon–baryon scattering amplitudes yield important constraints on baryon axial vector couplings and masses, as well as other static properties. Consistency of the large N_c limit requires exact cancellations amongst the tree diagram amplitudes at leading orders in N_c . These cancellations have been discussed explicitly in Refs. [2,4] for the simplest cases of baryon-pion scattering to a baryon plus one or two pions. The non-degeneracy of the baryon states in a given spin-flavor representation for finite N_c results in additional consistency conditions involving the baryon mass operator [3]. Interestingly, for baryons with two flavors of light quarks $N_F = 2$ and spin $J \sim \mathcal{O}(1)$, the leading contribution to the tree amplitude for baryon-pion scattering to a baryon and a single pion comes from terms which are first order in the baryon mass splittings [4].

In this paper, we generalize the prior analysis of baryon-pion scattering amplitudes by Dashen, Jenkins and Manohar [4]. We obtain new large- N_c consistency conditions for baryon axial vector couplings by studying tree scattering amplitudes to all orders in baryon mass splittings. We also obtain additional large- N_c consistency conditions for baryon vector current couplings. Lam and Liu [5] previously showed that the requisite cancellations of Ref. [4] occur in the degeneracy limit for tree scattering amplitudes containing an arbitrary number of single pion–baryon–baryon vertices. The analysis of this work extends this result to all orders in baryon mass splittings. As we have remarked, terms involving baryon mass splittings contribute to baryon-pion scattering amplitudes at leading order in N_c , so the demonstration

of consistency of the large- N_c limit for baryon-meson tree amplitudes is incomplete without the inclusion of baryon mass splittings.

The organization of this paper is as follows. In Sec. II, we give a brief description of large- N_c baryon spin-flavor symmetry and review baryon chiral perturbation theory in the $1/N_c$ expansion. In Sec. III, we study baryon-pion scattering amplitudes, and derive large- N_c consistency conditions to all orders in baryon mass splittings. We then compute the leading in N_c tree amplitude explicitly for a few specific examples, and comment upon the general case. We conclude in Sec. IV.

II. BARYON CHIRAL PERTURBATION THEORY IN THE $1/N_c$ EXPANSION

The lowest-lying baryons for large N_c are given by the completely symmetric spin-flavor representation of N_c quarks. Under $SU(2) \otimes SU(N_F)$, this $SU(2N_F)$ representation decomposes into a tower of baryon flavor representations with spins $J = \frac{1}{2}, \frac{3}{2}, \dots, \frac{N_c}{2}$. For two flavors of light quarks u and d , the baryon tower consists of (spin, isospin) representations with $I = J$, while for three flavors of quarks, the baryon flavor representations are considerably more complicated [4,6]. It is advantageous to concentrate on the baryon operators, rather than on the states, since baryon operators have a simple expansion in $1/N_c$ for arbitrary N_c .

The general form of the $1/N_c$ expansion of a QCD m -body quark operator acting on a single baryon state is given by

$$\mathcal{O}_{\text{QCD}}^{m\text{-body}} = N_c^m \sum_n c_n \frac{1}{N_c^n} \mathcal{O}_n, \quad (1)$$

where the \mathcal{O}_n , $0 \leq n \leq N_c$, are a complete set of linearly independent operator products which are of n^{th} order in the baryon spin-flavor generators, and the $c_n(1/N_c)$ are arbitrary unknown coefficients with an expansion in $1/N_c$ beginning at order unity. The large- N_c spin-flavor symmetry for baryons is generated by the baryon spin, flavor and spin-flavor operators J^i , T^a and G^{ia} which can be written for large, but finite, N_c as one-body quark operators acting on the N_c -quark baryon states:

$$\begin{aligned}
J^i &= q^\dagger \left(\frac{\sigma^i}{2} \otimes I \right) q, \\
T^a &= q^\dagger \left(I \otimes \frac{\lambda^a}{2} \right) q, \\
G^{ia} &= q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q,
\end{aligned} \tag{2}$$

where without loss of generality the baryon matrix elements of these operators can be taken as the values in the non-relativistic quark model.¹ The baryon spin-flavor operators satisfy the $SU(2N_F)$ algebra given in Table I. The operator basis \mathcal{O}_n for any QCD operator transforming according to a given spin \otimes flavor representation can be determined. Examples of $1/N_c$ expansions for baryon operators include the $1/N_c$ expansion of the baryon mass operator [3,7,8,4,6]

$$\begin{aligned}
\mathcal{M} &= \sum_{n=0}^{(N_c-1)/2} m_{2n} \frac{1}{N_c^{2n-1}} (J^2)^n \\
&= m_0 N_c \mathbb{1} + m_2 \frac{1}{N_c} J^2 + m_4 \frac{1}{N_c^3} J^4 + \dots,
\end{aligned} \tag{3}$$

and the $1/N_c$ expansion of the baryon axial vector current [6]

$$A^{ia} = a_1 G^{ia} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{ia} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{ia}. \tag{4}$$

In Eq. (4), the \mathcal{D}_n^{ia} are diagonal operators with nonzero matrix elements only between baryon states with the same spin, whereas \mathcal{O}_n^{ia} are purely off-diagonal operators with nonzero matrix elements only between baryon states of different spin. The explicit forms for these operators can be found in Ref. [6]. At the physical value $N_c = 3$, Eq. (3) reduces to

$$\mathcal{M} = m_0 N_c \mathbb{1} + m_2 \frac{1}{N_c} J^2, \tag{5}$$

and Eq. (4) reduces to

$$A^{ia} = a_1 G^{ia} + b_2 \frac{1}{N_c} J^i T^a + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{ia} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{ia}, \tag{6}$$

¹This convention is referred to as the quark representation in the literature, see Ref. [6].

where

$$\mathcal{D}_3^{ia} = \{J^i, \{J^j, G^{ja}\}\}, \quad (7)$$

$$\mathcal{O}_3^{ia} = \{J^2, G^{ia}\} - \frac{1}{2}\{J^i, \{J^j, G^{ja}\}\}. \quad (8)$$

The $1/N_c$ chiral Lagrangian describing the interactions of soft pions is formulated in terms of the field

$$\xi(x) = e^{i\Pi(x)/f}, \quad (9)$$

where $\Pi(x)$ is the nonet of Goldstone boson fields

$$\Pi(x) = \frac{\pi^a(x)\lambda^a}{2} + \frac{\eta'(x)I}{\sqrt{6}} \quad (10)$$

and $f \sim \mathcal{O}(\sqrt{N_c})$ is the pion decay constant. (The η' field is a Goldstone boson in the large- N_c limit because $U(1)_A$ is a symmetry that is broken only at order $1/N_c$ by the axial anomaly [9].) The $1/N_c$ chiral Lagrangian for matter fields depends on the ξ field through the vector and axial vector currents

$$\begin{aligned} \mathcal{V}^\mu &= \frac{1}{2} (\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi), \\ \mathcal{A}^\mu &= \frac{i}{2} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi). \end{aligned} \quad (11)$$

The Goldstone boson nonet vector and axial vector currents appearing in the $1/N_c$ baryon chiral Lagrangian are given by

$$\begin{aligned} \text{Tr}(\mathcal{V}^0 \lambda^a) &= \text{Tr} \left(\left(\frac{1}{2f^2} [\Pi, \partial^0 \Pi] - \frac{1}{4!} \frac{1}{f^4} [\Pi, [\Pi, [\Pi, \partial^0 \Pi]]] + \dots \right) \lambda^a \right) = \frac{i}{2f^2} f^{abc} \pi^b \partial^0 \pi^c + \dots, \\ \text{Tr} \left(\mathcal{V}^0 \frac{2I}{\sqrt{6}} \right) &= \text{Tr} \left(\left(\frac{1}{2f^2} [\Pi, \partial^0 \Pi] - \frac{1}{4!} \frac{1}{f^4} [\Pi, [\Pi, [\Pi, \partial^0 \Pi]]] + \dots \right) \frac{2I}{\sqrt{6}} \right) = 0, \end{aligned} \quad (12)$$

and

$$\begin{aligned} \text{Tr}(\mathcal{A}^i \lambda^a) &= \text{Tr} \left(\left(\frac{1}{f} \nabla^i \Pi - \frac{1}{3!} \frac{1}{f^3} [\Pi, [\Pi, \nabla^i \Pi]] + \dots \right) \lambda^a \right) = \frac{1}{f} \nabla^i \pi^a + \frac{1}{3! f^3} f^{abe} f^{cde} \pi^b \pi^c \nabla^i \pi^d + \dots, \\ \text{Tr} \left(\mathcal{A}^i \frac{2I}{\sqrt{6}} \right) &= \text{Tr} \left(\left(\frac{1}{f} \nabla^i \Pi - \frac{1}{3!} \frac{1}{f^3} [\Pi, [\Pi, \nabla^i \Pi]] + \dots \right) \frac{2I}{\sqrt{6}} \right) = \frac{1}{f} \nabla^i \eta', \end{aligned} \quad (13)$$

respectively. Notice that the $SU(3)$ singlet portion of the pion vector current vanishes identically, and that the singlet portion of the pion axial vector current is proportional to a single derivatively coupled η' .

The $1/N_c$ chiral Lagrangian for baryons in the baryon rest frame is given by [10]

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_{\text{hyperfine}} + \text{Tr} \left(\mathcal{A}^i \lambda^A \right) A^{iA} + \frac{1}{N_c} \text{Tr} \left(\mathcal{A}^i \frac{2I}{\sqrt{6}} \right) A^i + \dots, \quad (14)$$

where the ellipsis represents terms of higher order in the derivative and $1/N_c$ expansions as well as terms involving explicit chiral symmetry breaking by the quark mass matrix. In Eq. (14), the covariant derivative is equal to

$$\mathcal{D}^0 = \partial^0 \mathbb{1} + \text{Tr} \left(\mathcal{V}^0 \lambda^A \right) T^A = \partial^0 \mathbb{1} + \text{Tr} \left(\mathcal{V}^0 \lambda^a \right) T^a, \quad (15)$$

where $A = 1, \dots, 9$, and $\lambda^9 \equiv 2I/\sqrt{6}$, and the summation over the index A in the covariant derivative reduces to a summation over $a = 1, \dots, 8$ because the ninth component of the pion vector current vanishes identically. The leading $\mathcal{O}(N_c)$ singlet portion of the baryon mass has been removed from the $1/N_c$ baryon chiral Lagrangian by a phase redefinition of the baryon field as in heavy baryon chiral perturbation theory [11,12], so the $1/N_c$ baryon chiral Lagrangian depends only on baryon mass splittings. The baryon hyperfine mass operator is given by

$$\mathcal{M}_{\text{hyperfine}} = m_2 \frac{1}{N_c} J^2 + m_4 \frac{1}{N_c^3} J^4 + \dots. \quad (16)$$

The last two terms in Eq. (14) describe the axial couplings of a baryon to pions. The baryon axial vector current $A^{i9} \equiv A^i$ is defined in terms of Eq. (4) and the baryon one-body operators

$$\begin{aligned} G^{i9} &= \frac{1}{\sqrt{6}} J^i, \\ T^9 &= \frac{1}{\sqrt{6}} N_c \mathbb{1}. \end{aligned} \quad (17)$$

Nonet flavor symmetry of the pion–baryon–baryon axial vector couplings is broken explicitly by the last term in Eq. (14), which gives a nonet symmetry-breaking contribution to the

singlet baryon axial vector current A^i at relative order $1/N_c$. The baryon chiral Lagrangian has been written in the rest frame of the baryon for notational simplicity; it is straightforward to rewrite the Lagrangian in an arbitrary Lorentz frame in which the baryon travels with a fixed four-velocity v^μ .

$SU(3)$ -breaking baryon mass splittings arise from higher order terms in the chiral Lagrangian containing insertions of the quark mass matrix. The terms in the $1/N_c$ baryon chiral Lagrangian containing one power of $\mathcal{M}_q = \text{diag}(m_u, m_d, m_s)$ are given by

$$\mathcal{L}_{\text{baryon}}^{\mathcal{M}_q} = \text{Tr} \left(\left(\xi \mathcal{M}_q \xi + \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger \right) \frac{\lambda^a}{2} \right) \mathcal{H}^a + \frac{1}{N_c} \text{Tr} \left(\left(\xi \mathcal{M}_q \xi + \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger \right) \frac{I}{\sqrt{6}} \right) \mathcal{H}^0, \quad (18)$$

where $a = 3, 8, 9$. The baryon $1/N_c$ expansion of the QCD 1-body quark operator $(\bar{q}\lambda^a q)/2$ is given by

$$\mathcal{H}^a = \sum_{n=1}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^a, \quad (19)$$

where $\mathcal{D}_1^a = T^a$, $\mathcal{D}_2^a = \{J^i, G^{ia}\}$, and $\mathcal{D}_{n+2}^a = \{J^2, \mathcal{D}_n^a\}$. The baryon $1/N_c$ expansion of this scalar density reduces to

$$\mathcal{H}^a = b_1 T^a + b_2 \frac{1}{N_c} \{J^i, G^{ia}\} + b_3 \frac{1}{N_c^2} \{J^2, T^a\}, \quad (20)$$

for $N_c = 3$. Nonet flavor symmetry of the first term in Eq. (18) is broken by the second term which gives a nonet symmetry-breaking contribution to the singlet m_q -dependent baryon scalar density $\mathcal{H}^9 \equiv \mathcal{H}^0$ at relative order $1/N_c$.

Chiral perturbation theory for baryons in the $1/N_c$ expansion uses the vertices and propagators arising from the entire $1/N_c$ baryon chiral Lagrangian. In an arbitrary Lorentz frame, the baryon propagator is given by $i/(k \cdot v - \Delta)$ where

$$\Delta = M_I - M_{\text{ext}} \quad (21)$$

is the mass difference of the $I = \text{intermediate or internal baryon}$ and the external baryon. In heavy baryon chiral perturbation theory, Feynman diagrams are computed for soft pions and off-shell baryons with momenta $k \sim \mathcal{O}(1)$. Pion exchange only couples baryons with spins

differing by order unity. For baryons at the bottom of the spin tower with spin $J \sim \mathcal{O}(1)$, the baryon hyperfine mass splitting is $\mathcal{O}(1/N_c)$, while for baryons at the top of the spin tower with $J \sim \mathcal{O}(N_c)$, the baryon hyperfine mass splitting is $\mathcal{O}(1)$. The baryon mass splittings from terms dependent on quark masses are $\mathcal{O}(1)$, since the change in baryon flavor quantum numbers is $\mathcal{O}(1)$ for pion exchange. Thus, the baryon propagator can be expanded in a binomial expansion in baryon mass splittings

$$\left(\frac{i}{k \cdot v - \Delta}\right) = \left(\frac{i}{k \cdot v}\right) \sum_{n=0}^{\infty} \left(\frac{\Delta}{k \cdot v}\right)^n. \quad (22)$$

In the rest frame of the baryon, this reduces to

$$\left(\frac{i}{k^0 - \Delta}\right) = \left(\frac{i}{k^0}\right) \sum_{n=0}^{\infty} \left(\frac{\Delta}{k^0}\right)^n. \quad (23)$$

The binomial expansion of the baryon propagator is valid when the baryon mass difference is treated as a c -number. We will use this expansion in the next section to derive large- N_c consistency conditions for the baryon mass operator \mathcal{M} .

III. BARYON-PION SCATTERING AMPLITUDES

The amplitude for a baryon and pion to scatter to a final state consisting of a single baryon and $(n-1)$ pions is $\mathcal{O}(N_c^{1-n/2})$ by large- N_c power counting [13]. (For recent reviews of large- N_c power counting, consult Refs. [1,14]). The scattering amplitude is given by

$$\mathcal{A} = \mathcal{A}_{\text{vertex}} + \mathcal{A}_{\text{tree}} + \mathcal{A}_{\text{loop}}, \quad (24)$$

where $\mathcal{A}_{\text{vertex}}$ refers to the amplitude produced by contact n -meson-baryon-baryon vertex graphs; $\mathcal{A}_{\text{tree}}$ denotes the amplitude obtained from all other tree diagrams; and $\mathcal{A}_{\text{loop}}$ represents the amplitude obtained from all loop diagrams. Each of these terms is at most $\mathcal{O}(N_c^{1-n/2})$. Each vertex diagram is individually $\mathcal{O}(N_c^{1-n/2})$, so $\mathcal{A}_{\text{vertex}}$ is leading order. $\mathcal{A}_{\text{tree}}$ also is $\mathcal{O}(N_c^{1-n/2})$, but individual tree diagrams may grow with higher powers of N_c . For example, a tree diagram with n separate 1-pion-baryon-baryon vertices yields an amplitude which is $\mathcal{O}(N_c^{n/2})$, since each pion-baryon-baryon vertex is $\mathcal{O}(\sqrt{N_c})$. Only the sum

of all tree diagrams with n pion–baryon–baryon vertices is $\mathcal{O}(N_c^{1-n/2})$. Thus, the individual amplitudes for tree diagrams with n pion–baryon–baryon vertices must cancel exactly to $(n - 1)$ powers of N_c [5]. These exact cancellations must result from the operator structure of the tree amplitude. The loop amplitude $\mathcal{A}_{\text{loop}}$ is equal to

$$\mathcal{A}_{\text{loop}} = \sum_{L=1} \mathcal{A}_{\text{loop}}^{(L)}, \quad (25)$$

where $\mathcal{A}_{\text{loop}}^{(L)}$ denotes the scattering amplitude obtained from all diagrams containing L loops, $L \geq 1$. The amplitude $\mathcal{A}_{\text{loop}}^{(L)}$ is suppressed by a relative factor of $1/N_c^L$ compared to the leading in N_c amplitude, and so is order $N_c^{1-n/2-L}$. Thus, the leading in N_c portion of the scattering amplitude is equal to the amplitude of the contact vertices and the leading in N_c portion of the tree amplitude $\mathcal{A}_{\text{tree}}$. It is important to realize, however, that individual L -loop diagrams can grow with higher powers of N_c , so that exact cancellations result from the operator structure of the loop amplitude. Loop cancellations and the effect of baryon mass splittings on loop corrections are discussed in Ref. [15], which considers the renormalization of the baryon axial vector couplings at one-loop. We address the issue of tree diagram cancellations to all orders in baryon mass splittings in the remainder of this paper.

A. $B + \pi \rightarrow B' + \pi$

We begin by analyzing the simplest baryon-pion scattering process $B + \pi \rightarrow B' + \pi$ in large N_c , where π denotes one of the nine pseudo-Goldstone mesons π , K , η and η' . The scattering process is considered for soft pions with energies of order unity. The tree amplitude for this process is computed from the two tree diagrams displayed in Fig. 1 [2]. In the baryon rest frame,

$$\mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}'^j \times \left(\frac{A^{jb} A^{ia}}{(k^0 - M_I + M)} + \frac{A^{ia} A^{jb}}{(-k^0 - M_I + M')} \right), \quad (26)$$

where k^0 is the energy of the incoming pion, M is the mass of the initial baryon B , M' is the mass of the final baryon B' , M_I is the mass of the intermediate or internal baryon propagating

in the diagram, and the energy of the outgoing pion is $k^0 + M - M'$ by energy conservation. The incoming and outgoing pions have spin-flavor labels ia and jb , respectively, and couple to the baryon axial vector currents A^{ia} and A^{jb} . Each of the two tree diagrams in Fig. 1 contributes to the amplitude at $\mathcal{O}(N_c)$ since each pion-baryon-baryon vertex is $\mathcal{O}(\sqrt{N_c})$, but the total amplitude is at most $\mathcal{O}(1)$ by large- N_c power counting rules. Thus, the leading $\mathcal{O}(N_c)$ amplitudes of the two diagrams must cancel exactly. This cancellation requirement results in large- N_c consistency conditions. Large- N_c consistency conditions for the scattering amplitude are derived from Eq. (26) by expanding each baryon propagator in a power series in baryon mass differences over the pion energy,

$$\begin{aligned} \mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = & -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}'^j \times \left(\frac{1}{k^0} [A^{jb}, A^{ia}] \right. \\ & - \frac{(M - M_I)}{(k^0)^2} A^{jb} A^{ia} - \frac{(M' - M_I)}{(k^0)^2} A^{ia} A^{jb} \\ & \left. + \frac{(M - M_I)^2}{(k^0)^3} A^{jb} A^{ia} - \frac{(M' - M_I)^2}{(k^0)^3} A^{ia} A^{jb} + \dots \right), \end{aligned} \quad (27)$$

where the ellipsis refers to terms proportional to higher powers of baryon mass differences. The terms in Eq. (27) are proportional to c -number baryon mass differences and can be rewritten in terms of the baryon mass operator \mathcal{M} as

$$\begin{aligned} \mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = & -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}'^j \times \left(\frac{1}{k^0} [A^{jb}, A^{ia}] + \frac{1}{(k^0)^2} [A^{jb}, [\mathcal{M}, A^{ia}]] \right. \\ & \left. + \frac{1}{(k^0)^3} [A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] + \dots \right), \end{aligned} \quad (28)$$

or

$$\mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}'^j \times \left(\frac{1}{(k^0)} \sum_{n=0}^{\infty} \frac{1}{(k^0)^n} [A^{jb}, \underbrace{[\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ia}]]}_{n \text{ insertions}}] \right), \quad (29)$$

where n refers to the number of insertions of the baryon mass operator \mathcal{M} commuted with A^{ia} . Notice that each insertion of the baryon mass operator \mathcal{M} is accompanied by a commutator, so that the leading $\mathcal{O}(N_c)$ singlet piece of the baryon mass cancels out of the expression exactly and only the residual baryon mass operator gives a nonvanishing contribution. (For the moment, we neglect $SU(3)$ flavor symmetry-breaking, so the residual

baryon mass operator is equal to the hyperfine mass operator $\mathcal{M}_{\text{hyperfine}}$. When $SU(3)$ flavor breaking is not neglected, the residual baryon mass operator also includes baryon mass splittings due to quark masses.)

The large- N_c consistency conditions for $B + \pi \rightarrow B' + \pi$ scattering follow directly from Eq. (29) for $k^0 \sim \mathcal{O}(1)$. Each term with a different kinematic dependence on k^0 must individually satisfy the large- N_c power counting rule. The constraint that the tree amplitude be at most $\mathcal{O}(1)$ yields the large- N_c consistency conditions

$$\begin{aligned} [A^{jb}, A^{ia}] &\lesssim \mathcal{O}(N_c), \\ [A^{jb}, [\mathcal{M}, A^{ia}]] &\lesssim \mathcal{O}(N_c), \\ [A^{jb}, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] &\lesssim \mathcal{O}(N_c), \\ &\vdots \end{aligned} \tag{30}$$

since the factor of $1/f^2$ in the expression for the amplitude contains an implicit factor of $1/N_c$. The large- N_c consistency conditions in Eq. (30) can be written more compactly as

$$[A^{jb}, \underbrace{[\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ia}]] \dots]}_{n \text{ insertions}}] \lesssim \mathcal{O}(N_c), \tag{31}$$

for all n starting with $n = 0$.

If we restrict our attention to baryons with spins $J \sim \mathcal{O}(1)$, only a few of the above inequalities can be saturated and contribute to the scattering amplitude at leading order. This simplification occurs because operators with more powers of J can be neglected relative to the operators with fewer powers of J for baryons with $J \sim \mathcal{O}(1)$. For baryons with spin $J \sim \mathcal{O}(1)$, only the commutators $[A^{jb}, A^{ia}]$ and $[A^{jb}, [\mathcal{M}, A^{ia}]]$ contain an $\mathcal{O}(N_c)$ piece. The leading $\mathcal{O}(N_c)$ portion of these commutators is given explicitly by

$$\begin{aligned} [A^{jb}, A^{ia}] &= a_1^2 [G^{jb}, G^{ia}] + a_1 b_2 \left(\left[G^{jb}, \frac{1}{N_c} J^i T^a \right] + \left[\frac{1}{N_c} J^j T^b, G^{ia} \right] \right) + \dots \\ &= -i a_1^2 \left(\frac{1}{2} \epsilon^{ijk} d^{abc} G^{kc} + \frac{1}{4} \delta^{ij} f^{abc} T^c \right) - i \frac{1}{N_c} a_1 b_2 \epsilon^{ijk} (G^{kb} T^a + G^{ka} T^b) + \dots, \\ [A^{jb}, [\mathcal{M}, A^{ia}]] &= a_1^2 m_2 \left[G^{jb}, \left[\frac{1}{N_c} J^2, G^{ia} \right] \right] + \dots \\ &= a_1^2 m_2 \frac{1}{N_c} \left(\delta^{ij} \{ G^{\ell b}, G^{\ell a} \} - \{ G^{ib}, G^{ja} \} + \frac{1}{2} \epsilon^{ijk} f^{abc} J^k T^c \right) + \dots, \end{aligned} \tag{32}$$

where the ellipses denote terms which are subleading in $1/N_c$ compared to terms which have been retained. Thus, the leading $\mathcal{O}(1)$ portion of the tree amplitude for $B + \pi \rightarrow B' + \pi$ scattering is

$$\begin{aligned} \mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = & -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}^{j'} \times \left(\right. \\ & \frac{1}{(k^0)} \left(-ia_1^2 \left(\frac{1}{2} \epsilon^{ijk} d^{abc} G^{kc} + \frac{1}{4} \delta^{ij} f^{abc} T^c \right) - i \frac{1}{N_c} a_1 b_2 \epsilon^{ijk} (G^{kb} T^a + G^{ka} T^b) \right) \\ & \left. + \frac{1}{(k^0)^2} a_1^2 m_2 \frac{1}{N_c} \left(\delta^{ij} \{G^{\ell b}, G^{\ell a}\} - \{G^{ib}, G^{ja}\} + \frac{1}{2} \epsilon^{ijk} f^{abc} J^k T^c \right) \right) \end{aligned} \quad (33)$$

in the $SU(3)$ flavor symmetry limit.

The vertex amplitude contributes an additional $\mathcal{O}(1)$ piece from the 2-pion-baryon-baryon contact interaction shown in Fig. 2,

$$\mathcal{A}_{\text{vertex}}(B + \pi \rightarrow B' + \pi) = -\frac{1}{2f^2} (2k^0 + M - M') f^{abc} T^c. \quad (34)$$

Thus, the leading in $\mathcal{O}(1)$ amplitude is given by the sum of Eqs. (33) and (34) for baryons with spins $J \sim \mathcal{O}(1)$ in the limit of $SU(3)$ flavor symmetry.

Notice that for two flavors of light quark flavors $N_F = 2$ when (i) there is no d -symbol; (ii) the f -symbol reduces to the ϵ -symbol; and (iii) the flavor generator T^a reduces to the isospin generator I^a ; the leading $\mathcal{O}(1)$ portion of the tree amplitude for $B + \pi \rightarrow B' + \pi$ scattering for baryons with $J \sim I \sim \mathcal{O}(1)$ reduces to

$$\mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi) = -\frac{1}{f^2} \mathbf{k}^i \mathbf{k}^{j'} \times \left(\frac{1}{(k^0)^2} a_1^2 m_2 \frac{1}{N_c} \left(\delta^{ij} \{G^{\ell b}, G^{\ell a}\} - \{G^{ib}, G^{ja}\} \right) \right), \quad (35)$$

which originates from the commutator $[A^{jb}, [\mathcal{M}, A^{ia}]]$ that is first order in the baryon mass operator. The commutator $[A^{jb}, A^{ia}]$ that is zeroth order in the baryon mass operator yields a subdominant contribution to the tree amplitude, since the commutator of two axial vector baryon currents vanishes to two powers of N_c for $SU(4)$ spin-flavor symmetry [2]. The vertex contribution also is of subleading order for $N_F = 2$, so the leading $\mathcal{O}(1)$ scattering amplitude is equal to the tree amplitude in Eq. (35).

When explicit $SU(3)$ symmetry breaking is included, there are additional contributions to the scattering amplitude. For example, 2-pion-baryon-baryon vertices arising from terms

in the $1/N_c$ baryon chiral Lagrangian terms with insertions of the quark mass matrix contribute to the vertex scattering amplitude for $B + \pi \rightarrow B' + \pi$ scattering at zero energy. $SU(3)$ flavor symmetry breaking also appears in the tree amplitude through the baryon mass operator \mathcal{M} , which now contains flavor-dependent mass splittings. We will not evaluate these contributions explicitly.

B. $B + \pi \rightarrow B' + \pi + \pi$

The scattering process $B + \pi \rightarrow B' + \pi + \pi$ can be analyzed in a similar manner.

The scattering amplitude $\mathcal{A}(B + \pi \rightarrow B' + \pi + \pi)$ is $\mathcal{O}(1/\sqrt{N_c})$ by large- N_c power counting rules. The 3-meson-baryon-baryon vertex contribution to the scattering amplitude is explicitly $\mathcal{O}(1/\sqrt{N_c})$, and involves no subtle cancellations. The tree-diagram contribution arises from the six diagrams containing three 1-meson-baryon-baryon vertices displayed in Fig. 3 and the six diagrams containing one 1-meson-baryon-baryon vertex and one 2-meson-baryon-baryon vertex shown in Fig. 4. We consider these two sets of diagrams separately because the cancellations are disjoint.

The tree amplitude from the diagrams in Fig. 3 is given by

$$\begin{aligned} \mathcal{A}_{\text{tree}}^{\text{Fig. 3}}(B + \pi \rightarrow B' + \pi + \pi) = & \frac{i}{f^3} \mathbf{k}_\alpha^i \mathbf{k}_\beta^j \mathbf{k}_\gamma^k \times \left(\right. \\ & \frac{A^{kc} A^{jb} A^{ia}}{(k_\alpha^0 - M_1 + M)(k_\gamma^0 - M_2 + M')} + \frac{A^{jb} A^{kc} A^{ia}}{(k_\alpha^0 - M_1 + M)(k_\beta^0 - M_2 + M')} \\ & + \frac{A^{kc} A^{ia} A^{jb}}{(-k_\beta^0 - M_1 + M)(k_\gamma^0 - M_2 + M')} + \frac{A^{ia} A^{kc} A^{jb}}{(-k_\alpha^0 - M_2 + M')(-k_\beta^0 - M_1 + M)} \\ & \left. + \frac{A^{ia} A^{jb} A^{kc}}{(-k_\alpha^0 - M_2 + M')(-k_\gamma^0 - M_1 + M)} + \frac{A^{jb} A^{ia} A^{kc}}{(k_\beta^0 - M_2 + M')(-k_\gamma^0 - M_1 + M)} \right), \end{aligned} \quad (36)$$

where k_α^0 denotes the energy of the incoming pion with spin-flavor labels ia , k_β^0 denotes the energy of the outgoing pion with spin-flavor labels jb , and k_γ^0 denotes the energy of the outgoing pion with spin-flavor labels kc . Each of the pion energies is taken to be $\mathcal{O}(1)$. The masses M and M' are the masses of the initial baryon B and the final baryon B' , respectively, while M_1 and M_2 are the masses of the first and second intermediate baryons,

respectively. Energy conservation implies that $k_\alpha^0 + M = k_\beta^0 + k_\gamma^0 + M'$, so the energy k_α^0 can be eliminated from the expression using this relation. Each of the six diagrams produces a contribution of $\mathcal{O}(N_c^{3/2})$ to the amplitude, but the overall amplitude is only $\mathcal{O}(1/\sqrt{N_c})$, so exact cancellations must occur between the diagrams to two powers of N_c . Expanding the baryon propagators in powers of baryon mass differences over pion energies, and rewriting the expression using the baryon mass operator \mathcal{M} yields the lengthy expression given in Appendix A. Consistency of the large- N_c limit requires that the terms with different kinematic dependence must each be $\mathcal{O}(1/\sqrt{N_c})$. Taking into account the implicit $(1/\sqrt{N_c})^3$ dependence of the overall factor $1/f^3$, the linear combinations of commutators appearing in the expression must each be $\lesssim \mathcal{O}(N_c)$. Thus, the large- N_c consistency conditions obtained from the tree diagrams in Fig. 3 are:

$$[A^{kc}, [A^{ia}, A^{jb}]] \lesssim \mathcal{O}(N_c), \quad (37)$$

to zeroth order in baryon mass differences,

$$\begin{aligned} [A^{kc}, [A^{ia}, [\mathcal{M}, A^{jb}]]] &\lesssim \mathcal{O}(N_c), \\ [[\mathcal{M}, A^{kc}], [A^{ia}, A^{jb}]] &\lesssim \mathcal{O}(N_c), \end{aligned} \quad (38)$$

to first order in baryon mass differences, and

$$\begin{aligned} [[\mathcal{M}, [\mathcal{M}, A^{kc}]], [A^{ia}, A^{jb}]] &\lesssim \mathcal{O}(N_c), \\ [A^{kc}, [A^{ia}, [\mathcal{M}, [\mathcal{M}, A^{jb}]]]] &\lesssim \mathcal{O}(N_c), \\ [[\mathcal{M}, A^{kc}], [A^{ia}, [\mathcal{M}, A^{jb}]]] &\lesssim \mathcal{O}(N_c), \end{aligned} \quad (39)$$

to second order in baryon mass differences. In general, to n^{th} order in baryon mass differences, the consistency conditions restrict all possible commutators obtained from $[A^{kc}, [A^{ia}, A^{jb}]]$ with a total of n additional commutators of \mathcal{M} dressing the baryon axial vector currents A^{kc} and A^{jb} to be $\lesssim \mathcal{O}(N_c)$.

If we restrict our attention to baryons with spin $J \sim \mathcal{O}(1)$, not all of these commutators contribute to the $B + \pi \rightarrow B' + \pi + \pi$ scattering amplitude at leading order $\mathcal{O}(1/\sqrt{N_c})$. For

baryons with spin $J \sim \mathcal{O}(1)$, only the commutators of Eq. (37) and Eq. (38) and the last two commutators in Eq. (39) contain an $\mathcal{O}(N_c)$ piece. (The first commutator in Eq. (39) is necessarily suppressed in the $1/N_c$ expansion for baryons with spin $J \sim \mathcal{O}(1)$ since there are not enough commutators to get rid of all of the J 's coming from the baryon mass operators.) The leading $\mathcal{O}(N_c)$ portions of these commutators can be evaluated explicitly, but are rather lengthy and will be suppressed.

There are additional cancellations involving the 2-pion–baryon–baryon vertices occurring between the six tree diagrams displayed in Fig. 4. The amplitude produced by the diagrams in Fig. 4 is given by

$$\begin{aligned} \mathcal{A}_{\text{tree}}^{\text{Fig. 4}}(B + \pi \rightarrow B' + \pi + \pi) = & -\frac{i}{2f^3} \times \left(\right. \\ & \mathbf{k}_\alpha^i (k_\gamma^0 - k_\beta^0) f^{dbc} \left(\frac{T^d A^{ia}}{(k_\alpha^0 + M - M_I)} + \frac{A^{ia} T^d}{(-k_\alpha^0 + M' - M_I)} \right) \\ & - \mathbf{k}_\gamma^i (k_\alpha^0 + k_\beta^0) f^{dab} \left(\frac{T^d A^{ic}}{(-k_\gamma^0 + M - M_I)} + \frac{A^{ic} T^d}{(k_\gamma^0 + M' - M_I)} \right) \\ & \left. - \mathbf{k}_\beta^i (k_\alpha^0 + k_\gamma^0) f^{dac} \left(\frac{T^d A^{ib}}{(-k_\beta^0 + M - M_I)} + \frac{A^{ib} T^d}{(k_\beta^0 + M' - M_I)} \right) \right), \end{aligned} \quad (40)$$

where M_I is the mass of the intermediate baryon. The 2-pion–baryon–baryon vertex is $\mathcal{O}(1)$ whereas the 1-pion–baryon–baryon vertex is $\mathcal{O}(\sqrt{N_c})$, so each of the six diagrams is $\mathcal{O}(\sqrt{N_c})$. The large N_c power counting rule that the $B + \pi \rightarrow B' + \pi + \pi$ scattering amplitude is $\mathcal{O}(1/\sqrt{N_c})$ implies that the diagrams cancel exactly to one power in N_c . Expanding the terms in baryon mass differences over pion energies, and rewriting the expression in terms of the baryon mass operator \mathcal{M} yields

$$\begin{aligned} \mathcal{A}_{\text{tree}}^{\text{Fig. 4}}(B + \pi \rightarrow B' + \pi + \pi) = & -\frac{i}{2f^3} \times \left(\right. \\ & \mathbf{k}_\alpha^i \frac{(k_\gamma^0 - k_\beta^0)}{k_\alpha^0} f^{dbc} \left(\sum_{n=0}^{\infty} \left(\frac{1}{k_\alpha^0} \right)^n [T^d, [\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ia}] \dots]]] \right) \\ & + \mathbf{k}_\gamma^i \frac{(k_\alpha^0 + k_\beta^0)}{k_\gamma^0} f^{dab} \left(\sum_{n=0}^{\infty} \left(\frac{-1}{k_\gamma^0} \right)^n [T^d, [\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ic}] \dots]]] \right) \\ & \left. + \mathbf{k}_\beta^i \frac{(k_\alpha^0 + k_\gamma^0)}{k_\beta^0} f^{dac} \left(\sum_{n=0}^{\infty} \left(\frac{-1}{k_\beta^0} \right)^n [T^d, [\underbrace{\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ib}]}_{n \text{ insertions}}] \dots]]] \right) \right), \end{aligned} \quad (41)$$

where the terms in the summation have n commutators of \mathcal{M} with the baryon axial current.

The large- N_c consistency conditions for the diagrams in Fig. 4 are derived from Eq. (41) for pion energies of order unity. The constraint that the tree amplitude be at most $\mathcal{O}(1/\sqrt{N_c})$ yields the large- N_c consistency conditions

$$\begin{aligned} f^{dbc} [T^d, A^{ia}] &\lesssim \mathcal{O}(N_c), \\ f^{dbc} [T^d, [\mathcal{M}, A^{ia}]] &\lesssim \mathcal{O}(N_c), \\ f^{dbc} [T^d, [\mathcal{M}, [\mathcal{M}, A^{ia}]]] &\lesssim \mathcal{O}(N_c), \\ &\vdots \end{aligned} \tag{42}$$

or

$$f^{dbc} [T^d, \underbrace{[\mathcal{M}, [\mathcal{M}, \dots [\mathcal{M}, A^{ia}]]}_{n \text{ insertions}}]] \lesssim \mathcal{O}(N_c), \tag{43}$$

for all n starting with $n = 0$.

For baryons with spin $J \sim \mathcal{O}(1)$, only the first commutator in Eq. (42) can contribute to the scattering amplitude at leading order. Thus, the leading $\mathcal{O}(1/\sqrt{N_c})$ portion of the amplitude from the diagrams in Fig. 4 is given by

$$\begin{aligned} \mathcal{A}_{\text{tree}}^{\text{Fig. 4}} (B + \pi \rightarrow B' + \pi + \pi) \\ = -\frac{i}{2f^3} \times \left(\mathbf{k}_\alpha^i \frac{(k_\gamma^0 - k_\beta^0)}{k_\alpha^0} f^{dbc} [T^d, A^{ia}] - \mathbf{k}_\gamma^i \frac{(k_\alpha^0 + k_\beta^0)}{k_\gamma^0} f^{dab} [T^d, A^{ic}] - \mathbf{k}_\beta^i \frac{(k_\alpha^0 + k_\gamma^0)}{k_\beta^0} f^{dac} [T^d, A^{ib}] \right), \tag{44} \\ = \frac{1}{2f^3} \times \left(\mathbf{k}_\alpha^i \frac{(k_\gamma^0 - k_\beta^0)}{k_\alpha^0} f^{dbc} f^{dag} - \mathbf{k}_\gamma^i \frac{(k_\alpha^0 + k_\beta^0)}{k_\gamma^0} f^{dab} f^{dcg} - \mathbf{k}_\beta^i \frac{(k_\alpha^0 + k_\gamma^0)}{k_\beta^0} f^{dac} f^{dbg} \right) A^{ig}, \end{aligned}$$

for baryons with spin $J \sim \mathcal{O}(1)$, where the second equality follows because the baryon axial vector current transforms as a flavor adjoint under $SU(3)$ flavor symmetry. The amplitude Eq. (44) is now manifestly $\lesssim \mathcal{O}(1/\sqrt{N_c})$ since the matrix elements of the baryon axial vector current are $\lesssim \mathcal{O}(N_c)$ and the overall factor of $1/f^3$ contains an implicit factor of $(1/\sqrt{N_c})^3$.

The vertex amplitude contributes an $\mathcal{O}(1/\sqrt{N_c})$ piece from the contact 3-pion–baryon–baryon interaction given by

$$\begin{aligned} \mathcal{A}_{\text{vertex}}(B + \pi \rightarrow B' + \pi + \pi) = & \frac{i}{3!f^3} \left(-\mathbf{k}_\alpha^i \left(f^{bed} f^{cae} + f^{ced} f^{bae} \right) + \mathbf{k}_\beta^i \left(f^{ced} f^{abe} + f^{aed} f^{cbe} \right) \right. \\ & \left. + \mathbf{k}_\gamma^i \left(f^{aed} f^{bce} + f^{bed} f^{ace} \right) \right) A^{id}. \end{aligned} \quad (45)$$

Thus, the leading $\mathcal{O}(1/\sqrt{N_c})$ portion of the tree amplitude for $B + \pi \rightarrow B' + \pi + \pi$ scattering is given in the $SU(3)$ flavor symmetry limit by the sum of the leading $\mathcal{O}(1/\sqrt{N_c})$ part of the amplitude from the diagrams in Fig. 3, and Eqs. (44) and (45).

When explicit $SU(3)$ symmetry breaking is included, there are additional contributions to the scattering amplitude. $SU(3)$ flavor symmetry breaking appears in the tree amplitudes through the baryon mass operator \mathcal{M} , which now contains flavor-dependent mass splittings. In addition, there is a contribution to the 2-pion-baryon-baryon vertices in the diagrams in Fig. 4 from quark mass-dependent terms in the chiral Lagrangian. The 2-pion-baryon-baryon vertices from the $1/N_c$ baryon chiral Lagrangian linear in the quark mass matrix given in Eq. (18) are proportional to the baryon scalar density operator \mathcal{H}^a , $a = 3, 8, 9$. Large- N_c consistency conditions for these 2-pion-baryon-baryon vertices follow from the constraint that the scattering amplitude is $\lesssim \mathcal{O}(1/\sqrt{N_c})$. These additional large- N_c consistency conditions are:

$$f^{dbc} \left[\mathcal{H}^d, \underbrace{\left[\mathcal{M}, \left[\mathcal{M}, \dots \left[\mathcal{M}, A^{ia} \right] \dots \right] \right]}_{n \text{ insertions}} \right] \lesssim \mathcal{O}(N_c), \quad (46)$$

for all n starting with $n = 0$. Notice that Eq. (46) is obtained from Eq. (43) by replacing T^d by \mathcal{H}^d . The singlet $\mathcal{O}(N_c)$ piece of \mathcal{H}^a cancels out of the expression exactly, so that only the $a = 3$ and $a = 8$ components of \mathcal{H}^a give a nonvanishing contribution. Thus, the pion-nucleon sigma term does not contribute to the scattering amplitude. Similar large- N_c consistency conditions also can be found for terms in the $1/N_c$ baryon chiral Lagrangian with more insertions of the quark mass matrix.

C. $B + \pi \rightarrow B' + (n - 1)\pi$

The generalization to the scattering process $B + \pi \rightarrow B' + (n - 1)\pi$ for $n > 3$ is straightforward, although the expressions for the scattering amplitudes necessarily become

very lengthy. The large- N_c consistency conditions that follow from diagrams with n 1-pion-baryon-baryon vertices are that

$$\left[A^{i_n a_n}, \dots \left[A^{i_3 a_3}, \left[A^{i_2 a_2}, A^{i_1 a_1} \right] \right] \dots \right] \lesssim \mathcal{O}(N_c), \quad (47)$$

and that all multicommutators of n baryon axial vector currents with additional commutators of \mathcal{M} dressing the baryon axial vector currents in all possible ways are constrained to be at most $\mathcal{O}(N_c)$. Because of the multiplicative factor of $(1/f)^n$ in the amplitude, these large- N_c consistency conditions imply that the tree scattering amplitude is $\mathcal{O}(N_c^{1-n/2})$. Additional cancellations occur for diagrams with multimeson-baryon-baryon vertices, yielding large- N_c consistency conditions involving these vertices. All multipion-baryon-baryon vertices with an odd number of pions derive in the flavor symmetry limit from the pion axial vector current terms which are proportional to the baryon axial vector current operator. The large- N_c consistency conditions involving only vertices with odd numbers of pions therefore are of the form given in Eq. (47) where the number of axial vector currents is now less than n . Multipion-baryon-baryon vertices with an even number of pions derive in the flavor symmetry limit from the pion vector current terms which are proportional to the baryon flavor operator. Large- N_c consistency conditions involving one even-pion-baryon-baryon vertex and multiple odd-pion-baryon-baryon vertices are given by

$$\left[T^b, \left[A^{i_\ell a_\ell}, \dots \left[A^{i_2 a_2}, A^{i_1 a_1} \right] \dots \right] \right] \lesssim \mathcal{O}(N_c). \quad (48)$$

All multicommutators deriving from this multicommutator with additional commutators of \mathcal{M} dressing the baryon axial vector currents in all possible ways are constrained to be at most $\mathcal{O}(N_c)$. Diagrams with two even-pion-baryon-baryon vertices and multiple odd-pion-baryon-baryon vertices yield the large- N_c consistency conditions

$$\left[T^{b_2}, \left[T^{b_1}, \left[A^{i_\ell a_\ell}, \dots \left[A^{i_2 a_2}, A^{i_1 a_1} \right] \dots \right] \right] \right] \lesssim \mathcal{O}(N_c), \quad (49)$$

as well as all possible dressings of these multicommutators with commutators of \mathcal{M} being restricted to be at most $\mathcal{O}(N_c)$. The generalization to an arbitrary number of even-pion-baryon-baryon vertices is immediate. Other large- N_c consistency conditions follow from

terms in the $1/N_c$ baryon chiral Lagrangian with insertions of the quark mass matrix. Terms with no pions can be incorporated into the baryon mass operator, and appear in the multicommutators through \mathcal{M} . Terms with even numbers of pions can occur in place of some or all of the pion vector current couplings to baryons. The large- N_c consistency conditions involving these flavor symmetry breaking vertices are of the form

$$\left[T^{c_n}, \dots \left[\mathcal{T}^{c_1}, \left[\mathcal{H}^{b_m}, \dots \left[\mathcal{H}^{b_1}, \left[A^{i_\ell a_\ell}, \dots \left[A^{i_2 a_2}, A^{i_1 a_1}\right] \dots\right] \dots\right] \dots\right] \right] \lesssim \mathcal{O}(N_c). \quad (50)$$

All possible dressings of these multicommutators with arbitrary numbers of commutators of the baryon mass operator \mathcal{M} are constrained to be $\lesssim \mathcal{O}(N_c)$ as well.

IV. CONCLUSIONS

Exact cancellations occur in the tree amplitudes for baryon-pion scattering amplitudes at leading orders in N_c . As the number of pions involved in the scattering grows, the power in N_c of the cancellation also grows. In this paper, we derived the large- N_c consistency conditions for baryon-pion scattering processes to all orders in baryon mass splittings. The baryon mass splittings contribute to the scattering amplitudes through commutators of the baryon mass operator \mathcal{M} , so the $\mathcal{O}(N_c)$ singlet portion of the baryon mass cancels out of the amplitudes exactly. We showed that the leading in N_c portion of any baryon-pion scattering amplitude only requires the evaluation of terms to a finite order in baryon mass splittings for baryons with spins $J \sim \mathcal{O}(1)$. We explicitly computed the leading order in N_c scattering amplitude for the simplest processes $B + \pi \rightarrow B' + \pi$ and $B + \pi \rightarrow B' + \pi + \pi$. These cancellations were discussed previously in Ref. [4] to zeroth order in baryon mass splittings. The contribution to $B + \pi \rightarrow B' + \pi$ scattering at first subleading order in baryon mass splittings also was derived in Ref. [4], and was determined to be the dominant portion of the tree amplitude for $\pi^{\pm,0}$ scatterings. Additional large- N_c consistency conditions for baryon vector couplings, as well as for baryon couplings involving explicit $SU(3)$ symmetry breaking due to the quark mass matrix, were derived as well. There are many different

types of cancellations occuring in the tree scattering amplitudes, and the significance of the cancellations increases with the number of pions in the scattering process.

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APPENDIX A:

The contribution to $\mathcal{A}_{\text{tree}}(B + \pi \rightarrow B' + \pi + \pi)$ from the diagrams displayed in Fig. 3, written in terms of the baryon mass operator \mathcal{M} , is given by:

$$\begin{aligned}
\mathcal{A}_{\text{tree}}^{\text{Fig. 3}}(B + \pi \rightarrow B' + \pi + \pi) = & \frac{i}{f^3} \mathbf{k}_\alpha^i \mathbf{k}_\beta^j \mathbf{k}_\gamma^k \times \left(\right. \\
& - \frac{1}{(k_\beta^0 + k_\gamma^0)k_\gamma^0} [A^{kc}, [A^{ia}, A^{jb}]] - \frac{1}{(k_\beta^0 + k_\gamma^0)k_\beta^0} [A^{jb}, [A^{ia}, A^{kc}]] \\
& + \frac{1}{(k_\beta^0 + k_\gamma^0)^2 k_\gamma^0} ([A^{kc}, [A^{ia}, [\mathcal{M}, A^{jb}]]] + [[\mathcal{M}, A^{kc}], [A^{ia}, A^{jb}]]) \\
& + \frac{1}{(k_\beta^0 + k_\gamma^0)^2 k_\beta^0} ([A^{jb}, [A^{ia}, [\mathcal{M}, A^{kc}]]] + [[\mathcal{M}, A^{jb}], [A^{ia}, A^{kc}]]) \\
& + \frac{1}{(k_\beta^0 + k_\gamma^0)(k_\gamma^0)^2} [[\mathcal{M}, A^{kc}], [A^{ia}, A^{jb}]] + \frac{1}{(k_\beta^0 + k_\gamma^0)(k_\beta^0)^2} [[\mathcal{M}, A^{jb}], [A^{ia}, A^{kc}]] \\
& - \frac{1}{(k_\beta^0 + k_\gamma^0)^3 k_\gamma^0} ([[\mathcal{M}, [\mathcal{M}, A^{kc}]], [A^{ia}, A^{jb}]] + [A^{kc}, [A^{ia}, [\mathcal{M}, [\mathcal{M}, A^{jb}]]]]) \\
& \quad + 2 [[\mathcal{M}, A^{kc}], [A^{ia}, [\mathcal{M}, A^{jb}]]]) \\
& - \frac{1}{(k_\beta^0 + k_\gamma^0)^3 k_\beta^0} ([[\mathcal{M}, [\mathcal{M}, A^{jb}]], [A^{ia}, A^{kc}]] + [A^{jb}, [A^{ia}, [\mathcal{M}, [\mathcal{M}, A^{kc}]]]]) \\
& \quad + 2 [[\mathcal{M}, A^{jb}], [A^{ia}, [\mathcal{M}, A^{kc}]]]) \left. \right) \tag{A1}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{(k_\beta^0 + k_\gamma^0)^2 (k_\gamma^0)^2} \left([[\mathcal{M}, [\mathcal{M}, A^{kc}]], [A^{ia}, A^{jb}]] + [[\mathcal{M}, A^{kc}], [A^{ia}, [\mathcal{M}, A^{jb}]]] \right) \\
& -\frac{1}{(k_\beta^0 + k_\gamma^0)^2 (k_\beta^0)^2} \left([[\mathcal{M}, [\mathcal{M}, A^{jb}]], [A^{ia}, A^{kc}]] + [[\mathcal{M}, A^{jb}], [A^{ia}, [\mathcal{M}, A^{kc}]]] \right) \\
& -\frac{1}{(k_\beta^0 + k_\gamma^0)(k_\gamma^0)^3} [[\mathcal{M}, [\mathcal{M}, A^{kc}]], [A^{ia}, A^{jb}]] - \frac{1}{(k_\beta^0 + k_\gamma^0)(k_\beta^0)^3} [[\mathcal{M}, [\mathcal{M}, A^{jb}]], [A^{ia}, A^{kc}]] \\
& + \quad \dots \quad \left. \vphantom{\frac{1}{(k_\beta^0 + k_\gamma^0)(k_\gamma^0)^3}} \right),
\end{aligned}$$

where the ellipsis denotes terms with more powers of the baryon mass operator \mathcal{M} .

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TABLE I. $SU(2N_F)$ Commutation Relations

$$\begin{aligned}
[J^i, T^a] &= 0, \\
[J^i, J^j] &= i\epsilon^{ijk} J^k, & [T^a, T^b] &= if^{abc} T^c, \\
[J^i, G^{ja}] &= i\epsilon^{ijk} G^{ka}, & [T^a, G^{ib}] &= if^{abc} G^{ic}, \\
[G^{ia}, G^{jb}] &= \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2N_F}\delta^{ab}\epsilon^{ijk} J^k + \frac{i}{2}\epsilon^{ijk} d^{abc} G^{kc}.
\end{aligned}$$

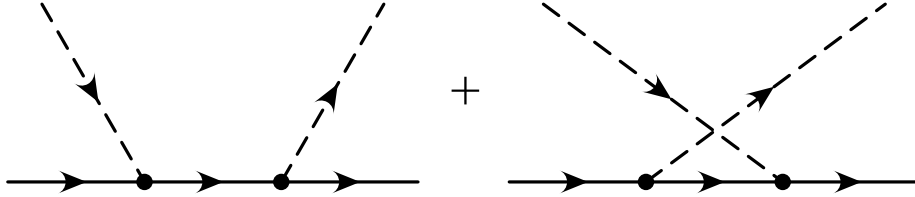


FIG. 1. Tree diagrams contributing to the $B + \pi \rightarrow B' + \pi$ scattering amplitude. Each individual diagram is $\mathcal{O}(N_c)$, but the sum of the two diagrams is $\mathcal{O}(1)$.

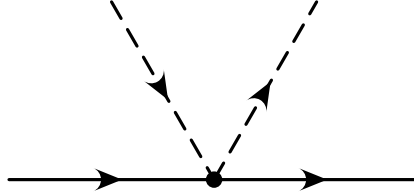


FIG. 2. Vertex contribution to the scattering amplitude for $B + \pi \rightarrow B' + \pi$. The vertex amplitude is $\mathcal{O}(1)$.

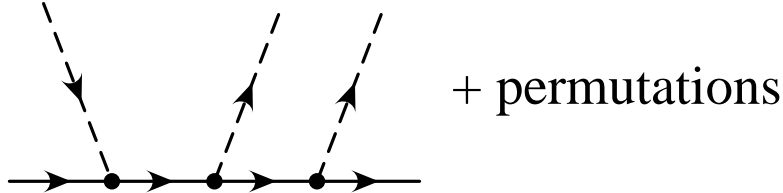


FIG. 3. Tree diagrams contributing to the $B + \pi \rightarrow B' + \pi + \pi$ scattering amplitude containing three $\mathcal{O}(\sqrt{N_c})$ pion-baryon-baryon vertices. Each individual diagram is $\mathcal{O}(N_c^{3/2})$, but the sum of the six diagrams is $\mathcal{O}(1/\sqrt{N_c})$.

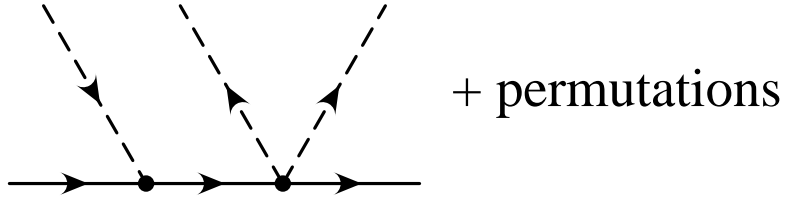


FIG. 4. Tree diagrams contributing to the $B + \pi \rightarrow B' + \pi + \pi$ scattering amplitude containing one $\mathcal{O}(\sqrt{N_c})$ pion-baryon-baryon vertex and one $\mathcal{O}(1)$ 2-pion-baryon-baryon vertex. Each individual diagram is $\mathcal{O}(\sqrt{N_c})$, but the sum of the six diagrams is $\mathcal{O}(1/\sqrt{N_c})$.